Examiners' Report
Principal Examiner Feedback

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Pearson Edexcel International Advanced Level
In Further Pure Mathematics F3 (WFM03)
Paper 01

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Examiners Report

General

This paper proved to be a good test of candidates' ability on the WFM03 content and plenty of opportunity was provided for them to demonstrate what they had learnt. Marks were available to candidates of all abilities. The questions that proved to be the most challenging were 5(b), 7 and 8.

Presentation was generally good and candidates often showed sufficient working to make their methods clear. In some cases candidates did not realise that earlier parts of questions were intended to help with later parts and so used more lengthy methods. This was particularly true in question 2 where candidates sometimes did not use the fact that **U** was the inverse of **T** and in question 7 where the identity in part (a) was not used to help with part (b).

Question 1

Most students knew how to tackle this question with generally only minor errors seen. The majority of students made a good attempt at part (i), with many gaining all 3 marks. Most students used the form of the integral given in the formula booklet but a significant minority used a trigonometric substitution. Errors sometimes occurred when substituting in the limits or with the coefficient of $\arctan\left(\frac{x}{4}\right)$ when integrating. A few scripts were seen where the students confused trigonometric functions and hyperbolic functions. Having arrived at $\arctan\left(\frac{x}{4}\right)$ they then applied the logarithmic definition for $\text{artanh}\left(\frac{x}{4}\right)$.

In part (ii) candidates sometimes had an extra factor of 2 and obtained $2\arcsin\left(\frac{2x}{3}\right)$ rather than $\arcsin\left(\frac{2x}{3}\right)$ whilst those who rearranged the denominator to $2\sqrt{\frac{9}{4}-x^2}$ were less likely to make this mistake. Having integrated and substituted the given limits, there were some confused attempts to find k including errors removing the arcsin and multiplying by $\frac{2}{3}$ instead of $\frac{3}{2}$. Again, though most candidates applied the formula directly from the formula booklet, a significant minority used a trigonometric substitution and were often successful.

Question 2

In part (a), the vast majority of candidates were able to set up at least two equations using

TU = **I** and then solve to find the correct values for *a*, *b* and *c*. Any loss of marks here was mainly due to errors occurring in forming the product of the matrices. A few candidates realised that using **UT** = **I** was equally valid and lead to simpler equations.

There was a mixed response to part (b) and it seemed to discriminate well. Many of the cohort used the given Cartesian equation of line 2 to form the parametric form and whilst most candidates obtained the correct parametric form, quite a few made a slip, treating $z + 2$ as $\frac{z+2}{0}$ rather than $\frac{z+2}{1}$. The need to use $\mathbf{T}^{-1}$ to find the equation of line 1 was recognised by the majority of the candidates but a significant minority did not realise that $\mathbf{T}^{-1}$ was **U** and instead attempted to find $\mathbf{T}^{-1}$ from first principals which lead to errors. Unfortunately some candidates used **T** in place of $\mathbf{T}^{-1}$ leading to a loss of marks. Having found the parametric form for the equation of line 1, many of the cohort then went on to correctly convert it into the required Cartesian form although a significant number of candidates stopped at the parametric form. Those attempting **Tx** = … often got in to trouble with the algebra.

Question 3

The majority of candidates found this question accessible and correctly answered parts (a), (b), (d) and (e). Very few candidates made any progress with part (c).

Nearly all candidates obtained the correct answers to part (a). The correct coordinates and equations were widely achieved although there were cases where only one of each was given and occasionally "directrices = " was seen rather than "$x =$ ".

In part (b) most candidates obtained suitable expressions for PS^2 and PM^2 though a few missed the y^2 when finding PS^2. Some students attempted to use $PS = ePM$ in this part of the question, not heeding the instruction that PS^2 should be an expression in terms of *e*, *x* and *y* and PM^2 an expression in terms of *e* and *x* only.

Part (c) caused a great deal of confusion with candidates not taking note of the demand of the question to use their expressions from part (b). They reverted to what they had been taught as a standard derivation of $b^2 = a^2\left(1 - e^2\right)$. Some who did use the results from part (b), forgot to square the e whilst others used $PS^2 = PM^2$. Most attempts were abandoned before obtaining an expression from which b^2 could be extracted.

Part (d) was well understood but several candidates forgot to disregard the negative value giving both $\pm\frac{1}{7}$ and surprisingly, a significant number chose the negative value.

Part (e) was well answered. Most were able to obtain the y-coordinate of P although a significant minority obtained 36 rather than 6. For the triangle area, most used the method outlined in the mark scheme although a few forgot to subtract $\frac{7}{2}$ and found a larger triangular area whilst a few found this and then tried to subtract the area of a smaller triangle.

Question 4

In part (a), the vast majority of the candidates multiplied the given eigenvector with the matrix **M** and deduced the correct value of the eigenvalue.

In part (b), as above, the majority of the cohort were able to find the eigenvector corresponding to the eigenvalue of -3 and any marks lost here were mainly due to sign errors or slips when solving the equations. Some candidates used 3 rather than -3.

In part (c), many of the candidates found the eigenvalue corresponding to the given eigenvector and thus correctly obtained the matrix **D** and the matrix **P**. The value of the third and final eigenvalue was occasionally found by constructing and solving the associated characteristic equation but a failure to match up these eigenvalues with their corresponding eigenvectors often resulted in a loss of marks. Occasionally, candidates did not, in forming the matrix **P**, normalise the eigenvectors which was perhaps due to not knowing what an orthogonal matrix was. A significant number of candidates did not realise that once all eigenvalues were known, the matrix **D** could just be written down and instead, resorted to calculating $\mathbf{P}^{\mathrm{T}}\mathbf{MP}$ with varying degrees of success.

Question 5

The majority of candidates were able to make progress with part (a). However, many candidates lost a mark as they did not show enough of the required working to justify the solution. Candidates should be reminded that when they are showing a given result, they need to show each step of their working. Some candidates used identities in terms of hyperbolic functions and did not prove the result from the exponential definitions as instructed in the question, thus losing all marks. Many attempts to “meet in the middle” failed to join up both sides sufficiently or did not make a final conclusion.

Part (b) proved difficult as most candidates were expecting it to work using integration by parts directly which almost invariably lead nowhere. Many candidates did not appreciate the scaffolding provided by part (a) and attempted splitting $\tanh^n(3x)$ into either $1\times\tanh^n(3x)$ or $\tanh(3x)\tanh^{n-1}(3x)$. Of those that did apply the correct $\tanh^2(3x)\tanh^{n-2}(3x)$ split and the result from part (a), many were able to achieve a fully correct proof, recognising the integral $\int \tanh(3x)\tanh^{n-1}(3x)\,dx$ could be found by inspection. Applying parts at this stage was only successful for a very small number of candidates.

Where candidates were able to complete part (b) they usually achieved some marks in (c). However, many had no attempt because they could not do part (b). It is worth noting that in this situation that some marks were available for using the given reduction formula in terms of *p*. The first mark was widely scored for attempting to use the reduction formula and although a lot of correct integrations of tanh 3*x* were seen, obtaining a correct I_1 was an obstacle for some and the $\frac{1}{3}$ was lost on occasion. Those who had scored the first two marks tended to get the next although there were some slips obtaining the “*c*” such that the last mark was not widely awarded.

Question 6

In part (a), many of the candidates carried out the well-rehearsed cross product procedure of finding the equation of a plane passing through three given points and full marks were often scored in this part. Some left their answer as a vector equation. A few candidates scored no marks because they used position vectors rather than obtaining two relevant vectors in the plane.

In part (b), although the values involved were not the usual expected integers, it was pleasing to see many candidates persist with their working and confidently dealt with obtaining a value of “λ” of $\pm\frac{85}{198}$. Some of the cohort, having found the equation of the line *DE* in parametric form, made algebraic slips in trying to evaluate λ and sometimes used an incorrect plane equation. Most did use a correct method to find the distance although the majority unnecessarily found the coordinates of *E* which led to an increased likelihood of numerical slips.

In part (c), the vast majority of the candidates identified the correct formula to find the volume of a tetrahedron and went on to construct and solve an equation in terms of *q* although a small number of candidates forgot the $\frac{1}{6}$ and lost 2 marks. Many candidates did not consider the case where V can be taken to be 12 or –12 and often, only one solution for q was found.

Question 7

In part (a), many candidates made progress towards given the result using the chain rule with many candidates also using implicit differentiation. The signs in the working were usually dealt with correctly. There was a small number of candidates who used incorrect identities in their working, or made sign errors and some did not show enough justification to gain the full three marks.

In part (b) most candidates were able to correctly differentiate $\coth x$ for the first mark. Candidates who worked in terms of $\cosh x$ often progressed to a correct quadratic equation and solved it correctly. Candidates that worked in terms of e^x often found the correct five term quartic but then made no further progress in solving it and stopped at that point. Very few candidates chose to work in terms of $\operatorname{sech} x$. Any errors in finding the derivative of $\coth x$ were very costly in terms of marks as even just a sign error led to a quadratic with solutions that were out of the range of $\cosh x$. Having obtained a solution to their quadratic in $\cosh x$, many assumed the given form of the answer and simply wrote the answer down in terms of their solution. As the demand of this question was to show the result, it was expected that candidates would substitute their solution into the logarithmic form of $\operatorname{arcosh} x$ or fully justify the solution by solving a quadratic in e^x.

Question 8

In part (a), the vast majority of the candidates were able to find either $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ and thus went on to obtain the required perimeter formula with correct values given for α and β although an incorrect value of β of 18 was seen at times. A few candidates incorrectly used the surface area of revolution formula and some candidates gave correct answers with no working and, as no differentiation was attempted, scored no marks.

In part (b), the majority of candidates scored the first mark for differentiating correctly to obtain $\frac{\mathrm{d}y}{\mathrm{d}u} = 4\cosh u$ although a few gave their answers as $\frac{\mathrm{d}y}{\mathrm{d}x} = 4\cosh u$ but this was condoned. The first method mark was also often scored for making a complete substitution although there were cases where $\frac{1}{4\cosh u}$ was seen in place of $4\cosh u$. For the integration, candidates who used appropriate hyperbolic identities often integrated correctly whilst those who converted to exponentials were more likely to make mistakes. The final two marks were only scored by the better candidates as many struggled to use the correct identities or correctly remove exponentials to obtain a numerical expression and some were unable to deal with substituting arsinh 3 into sinh 2*u* and could not achieve a numerical expression purely in terms of lns and constants. Often an incorrect upper limit was applied. A few who worked in exponentials left their answer in an unsimplified form with unprocessed squares of $3+\sqrt{10}$ and decimal answers were sometimes given. Only a small number of candidates achieved correct simplified exact value.